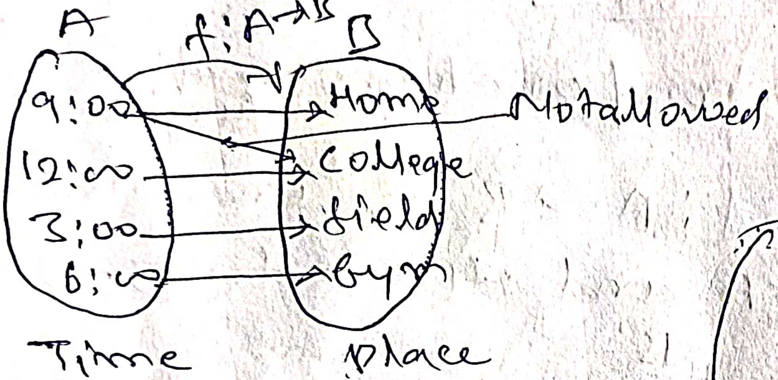
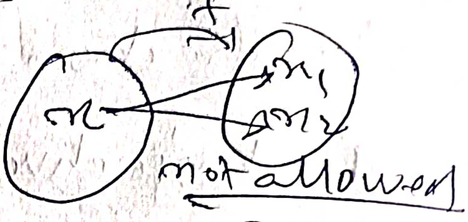
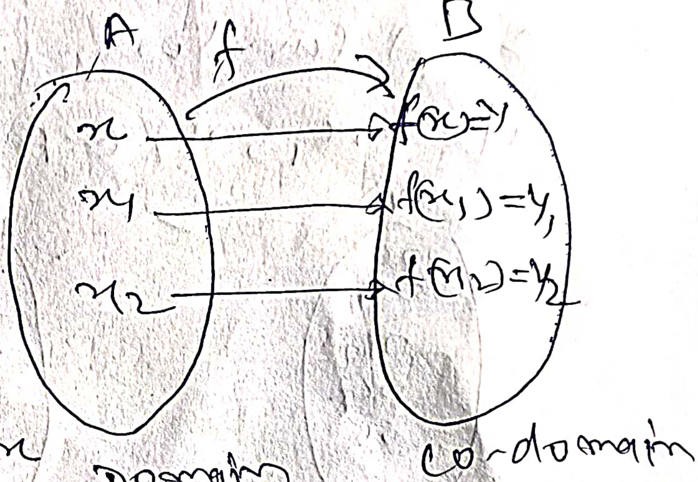
Function

Let A and B are non-empty sets.
A function $f : A \to B$ is a rule/mapping/relation from A to B such that
"each element of set A is related to exactly one element of set B".





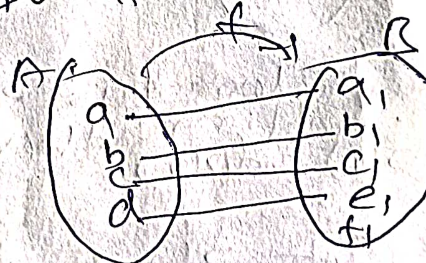


Note –

if $\boxed{x}$ $\xrightarrow{\text{then}}$ $\boxed{f(x) = y}$ image

Pre-image of y — image of x

where $x \in A$ & $y \in B$



Range of function –

Let $f : A \to B$ is a function
Then range of function is
$R(f)$ or Range(f) or $f(A)$
and defined as

$$R(f) = \{ f(x) ; x \in A \}$$

$R(f) = \{a_1, b_1, c_1, e_1\}$

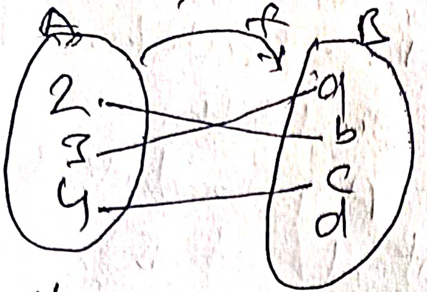
$\boxed{R(f) \subseteq B}$

Note: – Range of function is set of images.

Eg

$A = \{2, 3, 4\}$

$B = \{a, b, c, d\}$

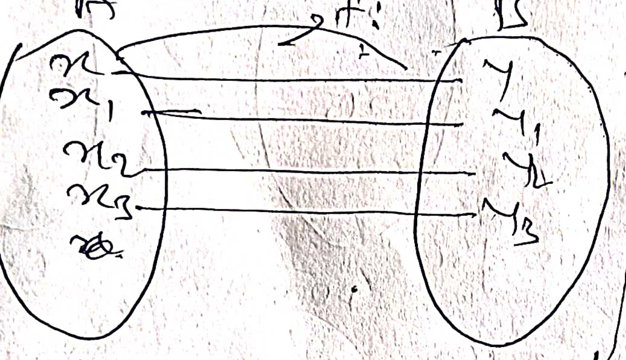


$\boxed{R(f) = \{a, b, c\}}$

Classification of function —

① Injective function / one to one mapping / function /

→ A function $f : A \to B$ is one-one function if $x_1, x_2 \in A$ and $\cancel{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$



$f(x) = y$

$f(x_1) = y_1$

$f(x_2) = y_2$

$f(x_3) = y_3$

If any two element are equal after mapping $f(x_1) = f(x_2)$ then

$\Rightarrow x_1 = x_2$

If before mapping elements are equal Then it is one one function

or

$x_1 \neq x_2$ → before mapping

$f(x_1) \neq f(x_2)$ → after "

Then function is one-one

Ex 1 Let $f: R \to R$ s.t

$f(x) = ax+b \quad \forall x \in R$

where R is real no.

Prove one-one function.

Solⁿ — Let $\boxed{x_1, x_2 \in R \ \& \ f(x_1) = f(x_2)}$

$\Rightarrow ax_1 + \cancel{b} = ax_2 + \cancel{b}$

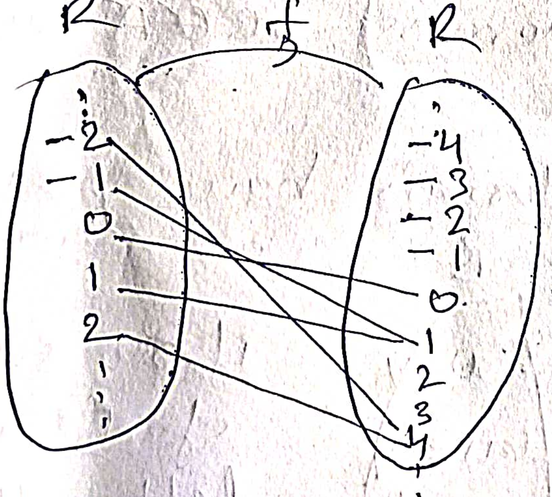
$\Rightarrow ax_1 = ax_2$

$\Rightarrow \boxed{x_1 = x_2}$

we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

This is only of one-one function.

Ex 2 — Let $f: R \to R$ defined by $f(x) = x^2 \quad \forall x \in R$



$f(x) = x^2$ $\quad f(2) = 4$

$f(-2) = (-2)^2 = 4$

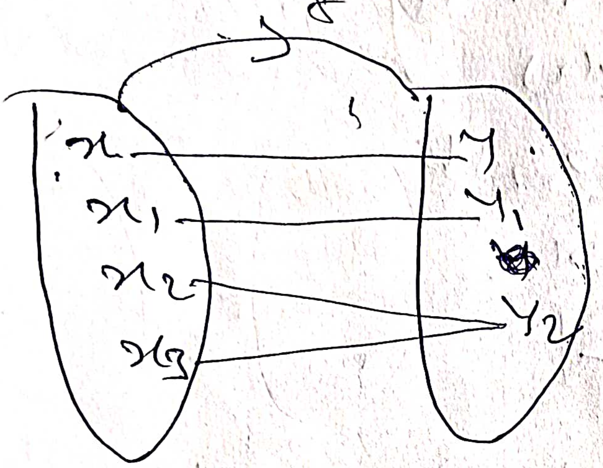
This is not one-one function (-2 & 2) map 4

$\left.\begin{array}{l} \Rightarrow f(2) = f(-2) \\ \Rightarrow 2 \neq -2 \end{array}\right\}$ so it is not one-one function.

Many-one function

A function $f: A \to B$ is a many-one if $x_1, x_2 \in A$ and $\underbrace{f(x_1) = f(x_2)}_{\text{if after mapping}} \Rightarrow \underbrace{x_1 \neq x_2}_{\text{if before mapping}}$

if after mapping & if before mapping The f is many-one



$f(x) = y$

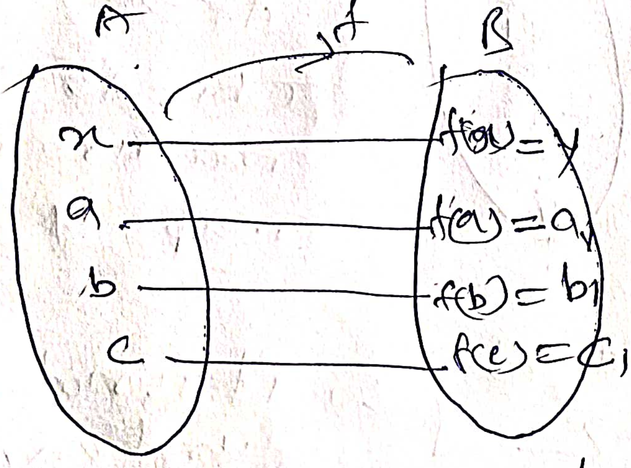
$f(x_1) = y_1$

$\boxed{\begin{array}{l} f(x_2) = y_2 \\ f(x_3) = y_2 \end{array}}$

Surjective function or on to function

A function $f: A \to B$ is on-to function if $\forall\, y \in B$ $\exists$ an element $x \in A$ such that $\boxed{f(x) = y}$

i.e. each element of codomain have preimage (at least one)



Domain co-domain

If no element of B are left without mapping then function is on-to funct

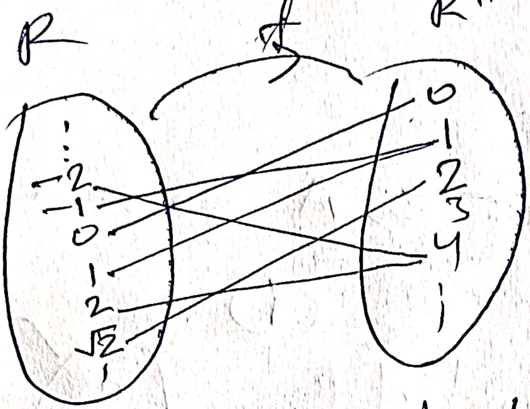
$\boxed{\forall\, y \in B \;\exists\, x \in A \text{ s.t } f(x) = y}$

Ex(1) $f: R \to R^+$ defined by $f(x) = x^2 \; \forall x \in R$

R → real no., R^+ → set of +ve real no.

Prove that it is on-to function?

Sol^n



$F(x) = x^2$

$f(-2) = 4$

$f(2) = 4$

all the element of R^+ are mapped so it is on-to function.

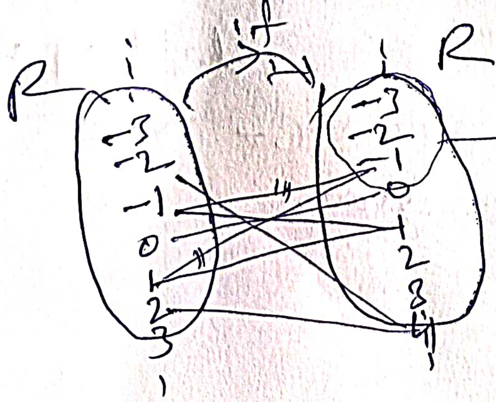
Note – Let $\forall\, y \in R^+ \; \exists$ an element ~~$\pm\sqrt{y} \in R$ s.t~~ $x \in R$ s.t. $f(x) = y$

$\Rightarrow x^2 = y$

$\Rightarrow \boxed{x = \pm\sqrt{y}}$

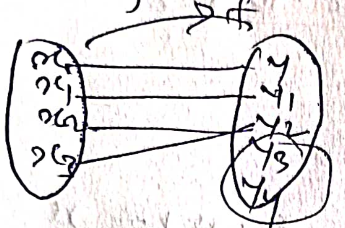
$\therefore$ $\boxed{\forall\, y \in R^+ \; \exists \text{ an element } \pm\sqrt{y} \in R \text{ s.t. } f(\pm\sqrt{y}) = (\pm\sqrt{y})^2 = y}$

Ex-2 Let $f: R \to R$ $f(x) = x^2 \; \forall x \in R$



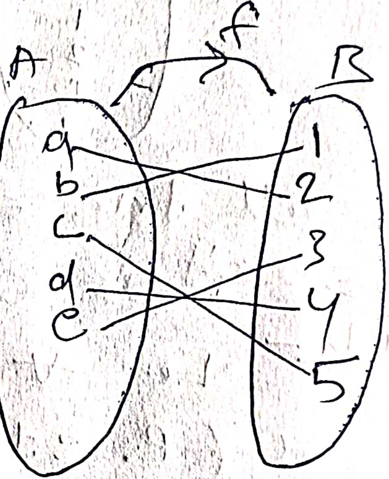
so it's not on-to function

Into function. — A function $f : A \to B$ is into function if $\exists$ at least one element $b \in B$ which has no preimage under f



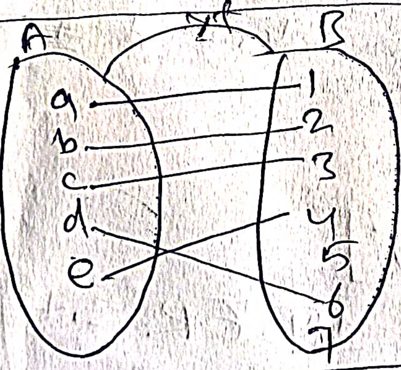
Bijective function $\Rightarrow$ A one-one-onto function

(i)



every element of B mapped so it is one-one and onto function.

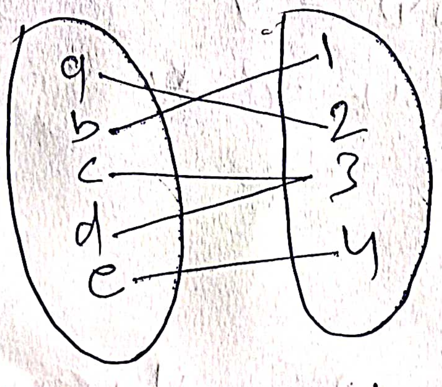
(ii)



two element of B left

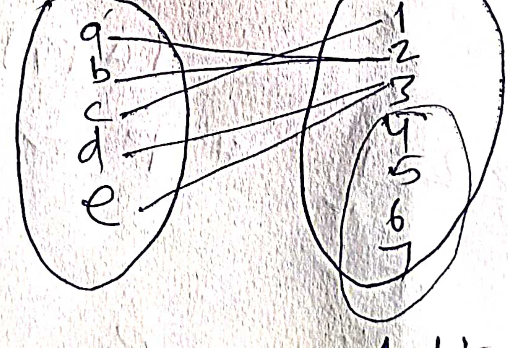
This is one-one and into function

(iii)



many one onto function.

(iv)



left 4, 5 & 7

Many one- into function.